Nonlocal Measurements in the Time-Symmetric Quantum Mechanics.

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Although nondemolition, reliable, and instantaneous quantum measurements of some nonlocal variables are impossible, demolition reliable instantaneous measurements are possible for all variables. It is shown that this is correct also in the framework of the time-symmetric quantum formalism, i.e. nonlocal variables of composite quantum systems with quantum states evolving both forward and backward in time are measurable in a demolition way. The result follows from the possibility to reverse with certainty the time direction of a backward evolving quantum state. Demolition measurements of nonlocal backward evolving quantum states require remarkably small resources. This is so because the combined operation of time reversal and teleportation of a local backward evolving quantum state requires only a single quantum channel and no transmission of classical information.

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Causality imposes constraints on nondemolition quantum measurements [1]. It allows measuring only certain classes of nonlocal variables [2]. Recently, it has been shown that it is possible to measure instantaneously an arbitrary nonlocal variable of two spin$-\frac{1}{2}$ particles [3] and of any composite system [4], provided we do not require nondemolition measurement. Such demolition measurement distinguishes with certainty between eigenstates of the nonlocal variable, but, contrary to the standard quantum measurement, it might destroy the measured eigenstate. The ability of performing such measurement provides physical meaning for nonlocal variables in the framework of relativistic quantum mechanics.

Recently, we witness significant developments of the time-symmetric formalism of quantum mechanics [5, 6] originated by Aharonov, Bergmann and Lebowitz [7]. This formalism contains, in addition to the standard quantum state evolving forward in time, the quantum state evolving backward in time from complete measurement performed in the future relative to the time in question. Moreover, in this framework one can define states evolving for one subsystem in one time direction and in the opposite time direction for another. Thus, one can define novel types of nonlocal variables, for which the eigenstates evolve backward in time or even in both time directions. In this work we show that such nonlocal variables are also measurable (in the sense of demolition measurements) and this result provides physical meaning for nonlocal variables and states of the time-symmetric quantum formalism.

Let us now state more precisely the problem we want to consider. The quantum system consists of two or more parts separated in space. We assume that we are equipped with measuring devices which allow measuring any local variable in each part of the system. We also have unlimited resources of entanglement, the quantum channels which connect the sites in which separate parts of the system are located. Given unlimited time, these resources would allow, via teleportation of the quantum states of the different parts to one location, a measurement of an arbitrary variable. However, we require *instantaneous* measurement and thus there is no time to complete the teleportation of the quantum states of the different parts. The meaning of "instantaneous" is that shortly after the time of measurement, there are permanent records at the locations of the various parts of the system, which jointly yield the value of the nonlocal variable. Examples of nonlocal observables are: variables with entangled eigenstates such as Bell operator, variables with product eigenstates which cannot be measured via measurements of local variables (which got the name *nonlocality without entanglement*) [8], etc. Some nonlocal variables are measurable in a stronger sense of standard nondemolition quantum measurements, but there are nonlocal variables which are not measurable in this sense [2]. This is why we consider demolition measurements which can be performed on all variables.

Let us recall the basic idea of nonlocal demolition measurements [4] in the case of a bipartite system where each part consists of several spin$-\frac{1}{2}$ particles. It consists of a sequence of "half teleportations" between the sites of the system, where, say, Alice and Bob are located. "Half teleportation" consists of the Bell measurement performed on the system and one particle from the EPR pair (the quantum channel). It does not include the transmission of the result of the Bell measurement and the appropriate correction of the state of the second particle of the EPR pair. Thus, "half-teleportation" does not have a minimal time for implementation.

We note that if a quantum state has a simple product form:

$$|\Psi\rangle = \Pi_i |s_i\rangle_i, \tag{1}$$

where $|s_i\rangle$ is either $|\uparrow_z\rangle$ or $|\downarrow_z\rangle$, then it can be unam-

biguously determined in a local measurement, even if it was first "half teleported" from another place. The information required for this includes the results of the spin measurements of the "half-teleported" state together with the results of the Bell measurements of the "half-teleportation" procedure. Indeed, the only possible effects of the second half of teleportation procedure are changes of the total phase and spin flips. The phase is irrelevant, and the effect of the spin flip can be corrected later, when the results of the Bell measurements arrive.

The procedure goes as follows. At the beginning, the state of Bob's part of the system is "half-teleported" to Alice. Alice assumes that the "half-teleportation" is, in fact, the full teleportation (there is a finite probability that the results of Bell measurements correspond to teleportation without correction) and transforms the eigenstates of the nonlocal variable to the orthogonal set of states in the form (1). Then, she "half-teleports" the transformed state to Bob. Bob knows the results of his teleportation Bell measurements and, if those were successful, he performs the spin measurements in the z basis. Together with the results of Alice teleportation Bell measurements these results yield the eigenstate of the measured variable. If Bob's first teleportation was not successful, he teleports the received state back to Alice in one of the numerous quantum channels according to the results of the Bell measurements in the first teleportation attempt. Now, Alice assumes that Bob's second teleportation was successful and unitarily transforms the outputs of all Bob's channels to the form (1) and then teleports each transformed state in separate channel to Bob. Every round adds finite probability for success, and the probability of success after many rounds converges to 1. The method can be generalized to any number of sites of a composite quantum system.

The method cannot be applied for measurements of backward evolving states by simple "time-reversal" of our operations. Indeed, in our procedure, Bob, starting from the second round, teleports the quantum state in a particular channel depending on the results of his previous Bell measurement. In the time reversed procedure, this corresponds to choosing the channel according to results of a measurement which has not been performed yet. Note, that standard von Neumann measurement *is* applicable for measurement of backward evolving quantum state. Here, the same procedure measures quantum states evolving forward and backward in time. The same holds also for nondemolition nonlocal quantum measurements [2]. Unfortunately, such measurements can be performed only for a very limited class of nonlocal variables. The question remains: Is it possible to measure (in a demolition way) an arbitrary variable for a backward evolving quantum state?

The answer is positive, and, moreover, we can measure in a demolition way any variable also for quantum states evolving in different directions at different locations. The solution is very simple: the quantum state evolving backward in time can be transformed to a state evolving forward in time. For quantum states evolving forward in time there is a measurement procedure which leads to a successful measurement with any desired probability. Therefore, there is a method for measuring a nonlocal variable for quantum state with parts evolving in arbitrary time directions.

Any quantum system with a finite number of states can be mapped onto a system of N spin$-\frac{1}{2}$ particles. So, all what we have to do is to change the time direction of a spin$-\frac{1}{2}$ particle. To this end, we perform a Bell measurement on the particle and an ancilla and then (if needed) local operations acting on the ancilla so as to change the state of the two particles to a singlet $\frac{1}{\sqrt{2}}(|\uparrow\rangle|\downarrow\rangle - |\downarrow\rangle|\uparrow\rangle)$. In fact, we can prepare the singlet in any other way too. We also have to ensure that there is no disturbance between the time of the Bell measurement and the time we receive the particle with the backward evolving state. Then, the ancilla obtains the time reversed state of the particle:

$$\alpha\langle\uparrow| + \beta\langle\downarrow| \rightarrow -\beta^*|\uparrow\rangle + \alpha^*|\downarrow\rangle. \tag{2}$$

When we perform such time reversal for all parts of the system evolving backward in time, the eigenstates of the nonlocal variable with parts of the system evolving backward in time are transformed to a well defined mutually orthogonal states evolving forward in time. Thus, we obtain a one to one correspondence between the eigenstates of the variable with parts evolving in different time directions and the eigenstates of a variable evolving forward in time. The latter we know how to measure in a demolition way, thus, we got a method for a demolition measurement of an arbitrary nonlocal variable.

Let us describe an example. In order to "prepare" an eigenstate of an operator corresponding to a quantum state evolving in part A forward in time and in part B backward in time consider two "crossed" nonlocal (in space and time) measurements, Fig.1:

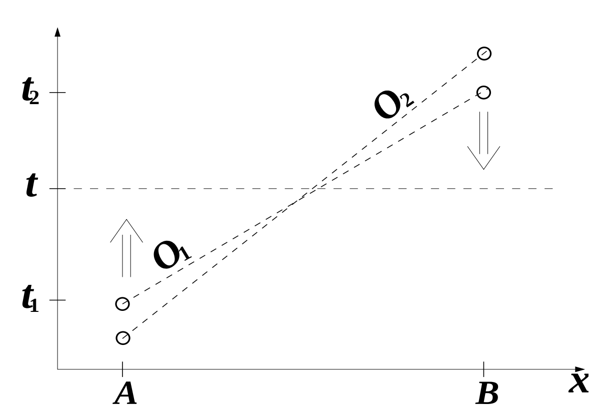

FIG. 1: The "preparation" of quantum state evolving forward in time in A and backward in time in B.

$$O_1 \equiv \sigma_z^A(t_1) - \sigma_z^B(t_2),$$
$$O_2 \equiv (\sigma_x^A(t_1-\epsilon) - \sigma_x^B(t_2+\epsilon))\mathrm{mod}4. \tag{3}$$

These measurements yield for time t, $t_1 < t < t_2$, one of the eigenstates of a nonlocal variable evolving forward in time in part A and backward in time in part B:

$$\begin{aligned}
O_1 &= 2: & &|\uparrow\rangle_A \ \ \langle\downarrow|_B, \\
O_1 &= -2: & &|\downarrow\rangle_A \ \ \langle\uparrow|_B, \\
O_1 &= 0, \ \ O_2 = 0: & &\frac{1}{\sqrt{2}}(|\uparrow\rangle_A \ \ \langle\uparrow|_B + |\downarrow\rangle_A \ \ \langle\downarrow|_B), \\
O_1 &= 0, \ \ O_2 = 2: & &\frac{1}{\sqrt{2}}(|\uparrow\rangle_A \ \ \langle\uparrow|_B - |\downarrow\rangle_A \ \ \langle\downarrow|_B).
\end{aligned} \tag{4}$$

Now, our time reversal operation (2) at time $t-\epsilon$ at part B transforms these eigenstates to

$$\begin{aligned}
&|\uparrow\rangle_A \ \ |\uparrow\rangle_B, \\
&|\downarrow\rangle_A \ \ |\downarrow\rangle_B, \\
&\frac{1}{\sqrt{2}}(|\uparrow\rangle_A \ \ |\downarrow\rangle_B - |\downarrow\rangle_A \ \ |\uparrow\rangle_B), \\
&\frac{1}{\sqrt{2}}(|\uparrow\rangle_A \ \ |\downarrow\rangle_B + |\downarrow\rangle_A \ \ |\uparrow\rangle_B)).
\end{aligned} \tag{5}$$

These are the eigenstates of a nonlocal variable evolving forward in time which cannot be measured in a nondemolition way [1], but can be measured in a demolition way.

Although the method explained above allows to measure any nonlocal variable, it requires unnecessary large resources. In order to make a reliable measurement of a forward evolving quantum state with parts located in several places, we need a huge amount of entanglement [4]. Amusingly, if in a particular location the quantum state evolves backward in time, the measurement can be performed in a simpler way, and a lot of entanglement resources can be saved. Our task is bringing the states of all parts to one location. The quantum state evolving backward in time can be moved to another place in one step which also includes the change of the direction of the time evolution of the state. All what we have to do is to prepare the EPR pair with our particle and an ancilla located in the place to which we want to move the state of the particle. When there is a large distance between the two places this might require a significant time, but we have this time since we can prepare the EPR pair in advance. If we cannot arrange in advance that one particle from the EPR pair is part of our post-selected system, we can always perform local swap operation between the member of the EPR pair and the post-selected particle. Thus, the backward evolving quantum state or, the state with only one part evolving forward in time, can be measured in a demolition way using very moderate resources: $N-1$ quantum channels for N-part system.

Until now we discussed quantum states evolving at each space-time point in a single time direction. In the framework of the time symmetric formalism of quantum mechanics [5], the complete description of a quantum system is given by a two-state vector $\langle\Phi| \ \ |\Psi\rangle$ such that there are both forward and backward evolving quantum states at each part of the system. The most general description of a composite quantum system of N parts is the *generalized two state vector* [9]:

$$\sum_i \alpha_{(i_1,..i_N,j_1,..j_N)} \prod_{n=1}^{N} \langle\Phi_{i_n}|_n \ \ |\Psi_{j_n}\rangle_n. \tag{6}$$

A generalized two-state vector might not be reducible to a two-state vector $\langle\Phi| \ \ |\Psi\rangle$. In this case, in order to "prepare" such a state at time t, it is necessary to have ancilla particles protected from the time of the the joint measurement of our system and the ancilla performed before t and until the joint post-selection measurement performed after time t. (Sufficiently large system can always be described by a two-state vector, like in standard quantum mechanics, sufficiently large system can always be described by a pure quantum state.)

The N-part system described by a generalized two-state vector is equivalent to $2N$-part system with quantum state in half of the parts evolving forward in time and in half of the parts evolving backward in time. Thus, preparing singlets with ancilla particles in the location of one of the parts of the system and their partners at the locations of the other parts, we can bring all backward evolving parts of the quantum state to our chosen location. From this point we proceed with the procedure of measuring nonlocal forward evolving state of N-part system [4].

The conceptual possibility of measuring an arbitrary variable with eigenstates which are nonlocal generalized two state-vectors provides justification of ascribing physical meaning to nonlocal two-state vectors. Since all nonlocal variables can be measured, the projection operators on all two-state vectors are measurable.

We now discuss further the concepts described above. What do we exactly mean by a "backward evolving state"? We need not assume that at present there is a state actually evolving backward in time from a future measurement, i.e., that the future is fixed. The formalism is helpful even in a pragmatic approach when we consider a scenario with post-selection: At a particular time we perform measurements on each member of an ensemble of quantum systems. Later, somebody performs another measurement and *post-selects* a subensemble of systems with a particular result of his measurement. There are no constrains on the post-selection measurement: any local or nonlocal variable can be measured, since there is no requirement that it will be instantaneous. All parts of the system can be brought to one place and then, by assumption, any measurement can be performed. Then, we

discard all the results for systems which were not post-selected and consider only systems which were successfully post-selected.

A basic requirement for a measurement, and in particular, for our demolition nonlocal measurement, is that it reliably distinguishes the eigenstates of an observed variable, i.e., our measurement and the post-selection should yield the same results. Then, it seems that a measurement of a variable for a backward evolving quantum state can be performed in a much simpler way: just *prepare* an eigenstate of this variable. Then, the post-selected ensemble will consists only of the cases in which the post-selection measurement yields the same result. Preparation of nonlocal state requires much less resources than its verification, since we can prepare the state of all parts of the system locally in one place and then bring them to the separate locations at the right time. (We assume through out the paper that the free Hamiltonian is zero. In practice we can always compensate for the free evolution.)

However, the preparation of a particular eigenstate is not a good enough verification measurement of a backward evolving state. What we ask from a quantum measurement is not only that it yields the correct value of the measured variable if the state is the eigenstate of this variable. Rather, we also need that if the state is a superposition of different eigenstates, then the measurement yields one of them with the appropriate probability. Thus, if the backward evolving state is a superposition of several eigenstates of the measured variable, then the preparation of one particular state yields incorrect probability.

Backward evolving state defines probability of a measurement at present only if there is no quantum state evolving forward in time from the measurements in the past. This is usually not the case (while it is a usual situation for the time-reversed situation in which forward evolving quantum state is considered since future is considered to be unknown). Thus the past of the quantum system has to be erased, it should be unknown. This can be done using collective measurements on the system and ancilla particles. After erasing the past we can prepare at random (using another ancilla) eigestates of the measured variable such that the probability will end up to be correct, but then, this procedure is probably not simpler than the one described above.

What allowed us to solve the problem of measurability of nonlocal variables with arbitrary directions of time evolution for various parts of the system is a simple, but apparently new result: *we can change the time direction with certainty from backward evolving quantum state to forward evolving quantum state.*

Note that this is not so if we try to transform forward to backward evolving state. In order to reverse the forward evolving state we need to obtain singlet as a result of Bell measurement performed on our particle and an ancilla. The probability for this is $\frac{1}{4}$. If the result is different, we cannot correct it to a singlet, since this operation depends on the results of Bell measurement, but has to be performed *before* the Bell measurement.

Although we do not consider here continuous systems (every real system can be approximated well as a system with a finite number of states) it is interesting to note that we can, conceptually, perform a time reversal of a backward evolving quantum state of a continuous system $\Psi(q)$ by preparing the original EPR state (the state of the Einstein, Podolsky, and Rosen paper [10]) for the particle and an ancilla:

$$|q - \tilde{q} = 0, \quad p + \tilde{p} = 0\rangle. \tag{7}$$

Then, the backward evolving quantum state of the particle will transform into a complex conjugate state of the ancilla:

$$\Psi(q) \rightarrow \Psi^*(\tilde{q}). \tag{8}$$

If the particle and the ancilla are located in different locations, then such operation is a combination of time reversal and teleportation of a backward evolving quantum state of a continuous variable [11].

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